### Solution

Given that the energy of the photon is 2.48 eV and converting this to joules, we get

$$p = \frac{E}{c} = \frac{(2.48 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{3.00 \times 10^8 \text{ m/s}} = 1.33 \times 10^{-27} \text{ kg} \cdot \text{m/s}.$$
29.33

#### Discussion

This value for momentum is the same as found before (note that unrounded values are used in all calculations to avoid even small rounding errors), an expected verification of the relationship p = E/c. This also means the relationship between energy, momentum, and mass given by  $E^2 = (pc)^2 + (mc)^2$  applies to both matter and photons. Once again, note that p is not zero, even when m is.

# **Problem-Solving Suggestion**

Note that the forms of the constants  $h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$  and  $hc = 1240 \text{ eV} \cdot \text{nm}$  may be particularly useful for this section's Problems and Exercises.

# **29.5 The Particle-Wave Duality**

We have long known that EM radiation is a wave, capable of interference and diffraction. We now see that light can be modeled as photons, which are massless particles. This may seem contradictory, since we ordinarily deal with large objects that never act like both wave and particle. An ocean wave, for example, looks nothing like a rock. To understand small-scale phenomena, we make analogies with the large-scale phenomena we observe directly. When we say something behaves like a wave, we mean it shows interference effects analogous to those seen in overlapping water waves. (See Figure 29.20.) Two examples of waves are sound and EM radiation. When we say something behaves like a particle, we mean that it interacts as a discrete unit with no interference effects. Examples of particles include electrons, atoms, and photons of EM radiation. How do we talk about a phenomenon that acts like both a particle and a wave?



Figure 29.20 (a) The interference pattern for light through a double slit is a wave property understood by analogy to water waves. (b) The properties of photons having quantized energy and momentum and acting as a concentrated unit are understood by analogy to macroscopic particles.

There is no doubt that EM radiation interferes and has the properties of wavelength and frequency. There is also no doubt that it behaves as particles $\hat{a}\epsilon$ "photons with discrete energy. We call this twofold nature the **particle-wave duality**, meaning that EM radiation has both particle and wave properties. This so-called duality is simply a term for properties of the photon analogous to phenomena we can observe directly, on a macroscopic scale. If this term seems strange, it is because we do not ordinarily observe details on the quantum level directly, and our observations yield either particle *or* wavelike properties, but never both simultaneously.

Since we have a particle-wave duality for photons, and since we have seen connections between photons and matter in that both have momentum, it is reasonable to ask whether there is a particle-wave duality for matter as well. If the EM radiation we once thought to be a pure wave has particle properties, is it possible that matter has wave properties? The answer is yes. The consequences are tremendous, as we will begin to see in the next section.

# **Quantum Wave Interference**

When do photons, electrons, and atoms behave like particles and when do they behave like waves? Watch waves spread out and interfere as they pass through a double slit, then get detected on a screen as tiny dots. Use quantum detectors to explore how measurements change the waves and the patterns they produce on the screen. <u>Click to open media in new browser</u>. (https://phet.colorado.edu/en/simulation/legacy/quantum-wave-interference)

# **29.6 The Wave Nature of Matter** De Broglie Wavelength

In 1923 a French physics graduate student named Prince Louis-Victor de Broglie (1892–1987) made a radical proposal based on the hope that nature is symmetric. If EM radiation has both particle and wave properties, then nature would be symmetric if matter also had both particle and wave properties. If what we once thought of as an unequivocal wave (EM radiation) is also a particle, then what we think of as an unequivocal particle (matter) may also be a wave. De Broglie's suggestion, made as part of his doctoral thesis, was so radical that it was greeted with some skepticism. A copy of his thesis was sent to Einstein, who said it was not only probably correct, but that it might be of fundamental importance. With the support of Einstein and a few other prominent physicists, de Broglie was awarded his doctorate.

De Broglie took both relativity and quantum mechanics into account to develop the proposal that *all particles have a wavelength*, given by

$$\lambda = \frac{h}{p}$$
 (matter and photons), 29.34

where *h* is Planck's constant and *p* is momentum. This is defined to be the **de Broglie wavelength**. (Note that we already have this for photons, from the equation  $p = h/\lambda$ .) The hallmark of a wave is interference. If matter is a wave, then it must exhibit constructive and destructive interference. Why isn't this ordinarily observed? The answer is that in order to see significant interference effects, a wave must interact with an object about the same size as its wavelength. Since *h* is very small,  $\lambda$  is also small, especially for macroscopic objects. A 3-kg bowling ball moving at 10 m/s, for example, has

$$\lambda = h/p = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})/[(3 \text{ kg})(10 \text{ m/s})] = 2 \times 10^{-35} \text{ m}.$$

29.35

This means that to see its wave characteristics, the bowling ball would have to interact with something about  $10^{-35}$  m in size—far smaller than anything known. When waves interact with objects much larger than their wavelength, they show negligible interference effects and move in straight lines (such as light rays in geometric optics). To get easily observed interference effects from particles of matter, the longest wavelength and hence smallest mass possible would be useful. Therefore, this effect was first observed with electrons.

American physicists Clinton J. Davisson and Lester H. Germer in 1925 and, independently, British physicist G. P. Thomson (son of J. J. Thomson, discoverer of the electron) in 1926 scattered electrons from crystals and found diffraction patterns. These patterns are exactly consistent with interference of electrons having the de Broglie wavelength and are somewhat analogous to light interacting with a diffraction grating. (See Figure 29.21.)

### **Connections: Waves**

All microscopic particles, whether massless, like photons, or having mass, like electrons, have wave properties. The relationship between momentum and wavelength is fundamental for all particles.

De Broglie's proposal of a wave nature for all particles initiated a remarkably productive era in which the foundations for